Superradiant instabilities in astrophysical systems

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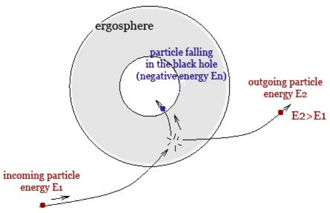
Outline

Motivation

Superradiance effect

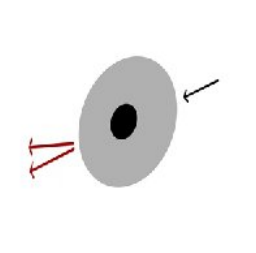
- Penrose process
  (Penrose '69, Christodoulou '70)
  - scattering of particles off Kerr BH

  ⇒ reduction of BH mass



- superradiant scattering
  (Misner '72, Zeldovitch '71)
  - scattering of wave packet off Kerr BH
  - superradiance condition

$$\omega < m\Omega_H = m\frac{a}{2Mr_+}$$

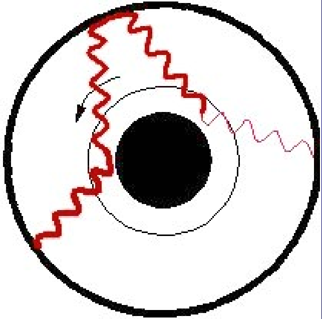


⇒ extraction of energy and angular momentum off BH
⇒ amplification of energy and angular momentum of wave packet

Superradiance instability

"black hole bomb" (Press & Teukolsky '72, Zeldovich '71, Cardoso et al '04)

- consider Kerr BH surrounded by mirror
- consider field with $\omega < m\Omega_H$
  $\Rightarrow$ superradiant scattering
- subsequequent amplification of superradiant modes

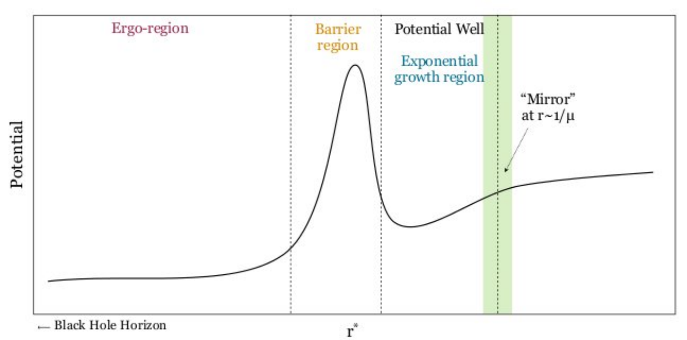


$\Rightarrow$ exponential growth of modes
$\Rightarrow$ instability due to superradiant scattering

Superradiance instability in physical systems

natural mirror provided by

- anti-de Sitter spacetimes
- massive fields with mass coupling $M\mu$
  (Damour et al. '76, Detweiler '80, Zouros & Eardley '79)



Arvanitaki & Dubovsky '11

Superradiance instability in physical systems

growth rate of massive scalar fields

- Detweiler '80: $M\mu << 1$

$$\tau \sim 24 \left(\frac{a}{M}\right)^{-1} (M\mu)^{-9} \left(\frac{GM}{c^3}\right)$$

- Zouros & Eardley '79 $M\mu >> 1$

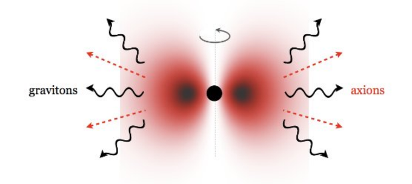
$$\tau \sim 10^7 \exp(1.84 M\mu) \left(\frac{GM}{c^3}\right)$$

- for astrophysical BHs and known particles: $M\mu \sim 10^{18}$
  $\Rightarrow$ insignificant for astrophysical systems?

Superradiance instability in physical systems

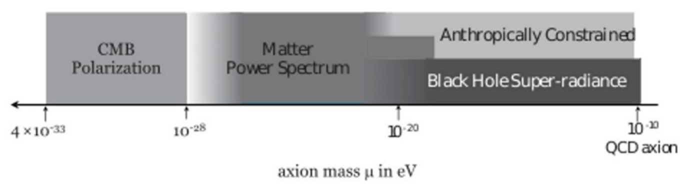
- most promising mass range: $M\mu \sim 1$
- ultralight bosons proposed by string theory compactifications: axions (Arvanitaki & Dubovsky '10)

- formation of bosonic bound states around astrophysical BHs
- gravitational wave emission
- "bosenova"-like particle bursts (Yoshino & Kodama '12)



(Arvanitaki & Dubovsky '11)

Superradiance instability in physical systems



(Kodama & Yoshino '11)

- landscape of ultralight axions $\Rightarrow$ "string axiverse"
- bosonic fields with $M\mu \sim 10^{-22}$ as dark matter candidates
- small, primordial BHs with $M \sim 10^{-18} M_{\odot}$
- bosonic cloud around SMBHs ($M \sim 10^{9} M_{\odot}$) if $10^{-21} \leq M\mu \leq 10^{-16}$
  $\Rightarrow$ probe of photon mass (upper bound $\mu_{\gamma} \sim 10^{-18}$ (Nakamura et al.'10))

Massive scalar field

Massive scalar fields - recent results

- Klein-Gordon equation

$$(\Box - \mu^2)\psi = 0\,, \quad \text{with} \quad \psi = \exp(im\phi - i\omega t) S_{lm}(\theta) R_{lm}(r)$$

- QNMs (Dolan '07, Cardoso & Yoshida '05, Berti et al '09)

$M\mu$	a/M	$M\omega_{11}$ $(n=0)$	$M\omega_{11}$ $(n=1)$
0.00	0.00	$0.2929 - i0.09766$	$0.2645 - i0.3063$
0.00	0.99	$0.4934 - i0.03671$	$0.4837 - i0.09880$
0.42	0.00	$0.4075 - i0.001026$	$0.4147 - i0.0004053$
0.42	0.99	$0.4088 + i1.504 \cdot 10^{-7}$	$0.4151 + i5.364 \cdot 10^{-8}$

- Tails
  - massless field (Price '71, Leaver '86, Ching et al '95)

$$\psi \sim t^{-2l-3}$$

  - massive fields (Koyama & Tomimatsu '01,'02, Burko & Khanna '04)

$$\psi \sim t^{-l-3/2} \sin(\mu t)\,, \quad \text{at intermediate times}$$

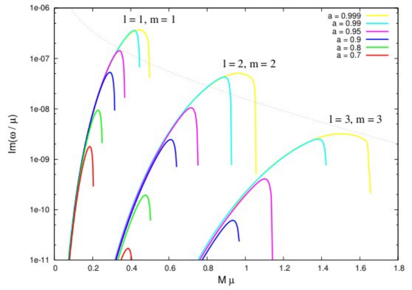
$$\psi \sim t^{-5/6} \sin(\mu t)\,, \quad \text{at very late times}$$

Massive scalar fields - recent results

- bound states: maximum instability growth rate for (Dolan '07, Cardoso & Yoshida '05)

$$l = m = 1\,, \quad a/M = 0.99\,, \quad M\mu = 0.42: \quad \frac{1}{\tau} = \omega_I \sim 1.5 \cdot 10^{-7} \left(\frac{GM}{c^3}\right)^{-1}$$

- numerical results:
  - Strafuss & Khanna '05: $M\omega_I \sim 2 \cdot 10^{-5}$ (Gaussian initial data)
  - Kodama & Yoshino '12: $M\omega_I \sim 3.2 \cdot 10^{-7}$ (bound state initial data)



Dolan '07

Massive scalar fields - Code setup

goal: study time evolution of massive scalar field in Kerr background

- Kerr background in Kerr-Schild coordinates $\rightarrow$ excision of BH region
- Klein-Gordon equation $(\Box - \mu^2)\psi = 0$ as $3+1$ time evolution problem

$$\begin{aligned} d_t\psi &= -\alpha\Pi \\ d_t\Pi &= -\alpha(D^iD_i\psi - \mu^2\psi - K\Pi) - D^i\alpha D_i\psi \end{aligned}$$

- initial data:
  - Gaussian wave packet with $r_0 = 12M$, $w = 2M$
  - quasi-bound state
- 4^{th} finite differences in space, 4^{th} order Runge-Kutta time-integrator
- extraction of scalar field at fixed r_{ex}, mode decomposition

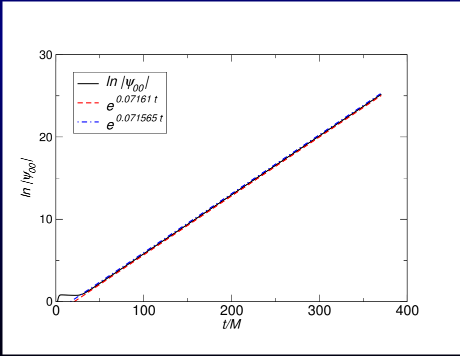
$$\psi_{lm}(t) = \int d\Omega\psi(t,\theta,\phi)Y^*_{lm}(\theta,\phi)$$

Code tests I - space dependent mass coupling

- consider spherically symmetric scalar field with *unphysical* mass

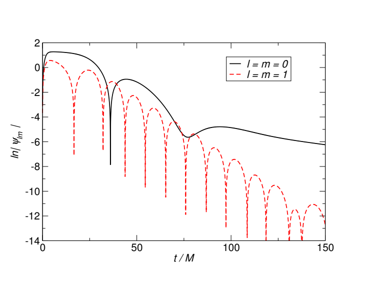
$$(M\mu)^2 = -\frac{10}{r^4}$$

- theoretical prediction: $\psi \sim \exp(0.071565\,t)$
- numerical result: $\psi \sim \exp(0.07161\,t) \Rightarrow$ agreement within 0.06%

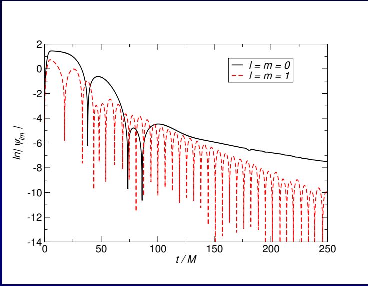


Code tests II - massless scalar fields

$a/M = 0$



$a/M = 0.99$



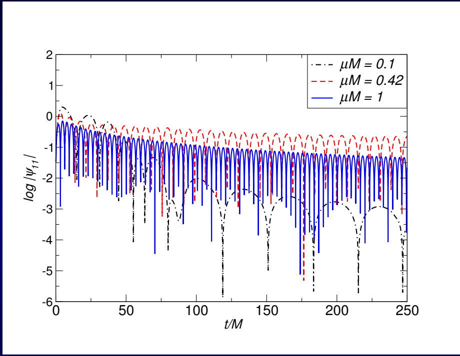
- QNM frequencies

$$\begin{array}{lll} a/M = 0.0 & M\omega_{11} = 0.294 - \imath 0.096 & (0.2929 - \imath 0.09766) \\ a/M = 0.99 & M\omega_{11} = 0.493 - \imath 0.0368 & (0.4934 - \imath 0.03671) \end{array}$$

- tail: $\psi_{00} \sim t^{-3.04}$ $(\psi_{00} \sim t^{-3})$
- numerical error: $\Delta\psi_{00}/\psi_{00} \leq 3\%$, . $\Delta\psi_{11}/\psi_{11} \leq 8\%$

Massive scalar fields in Schwarzschild

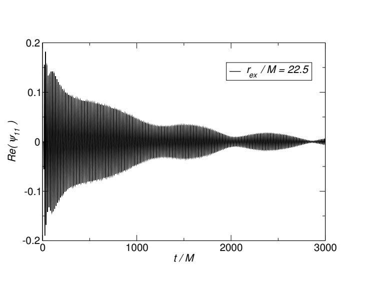
consider mass coupling $M\mu = 0.1, 0.42, 1$

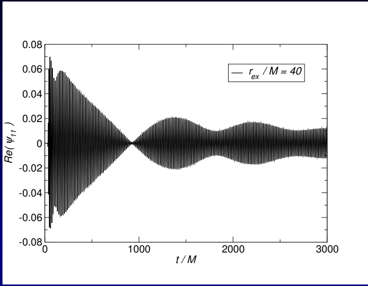


- tails in agreement with (Koyama & Tomimatsu '02, Burko & Khanna '04):

$$M\mu = 0.1 \quad M\omega_{11} = 0.293 - \imath 0.036 \quad \psi_{11} \sim t^{-2.543} \sin(0.1t)$$
$$M\mu = 1.0 \quad M\omega_{11} = 0.965 - \imath 0.0046 \quad \psi_{11} \sim t^{-0.873}$$

Massive scalar field with $M\mu = 0.42$ in Schwarzschild





- dependent on location of measurement
- beating of modes:
  - similar frequency $M\omega_R$ of fundamental and overtone mode
    $\omega_{R,n} = \omega_{R,0} + \delta,\ \delta << 1$

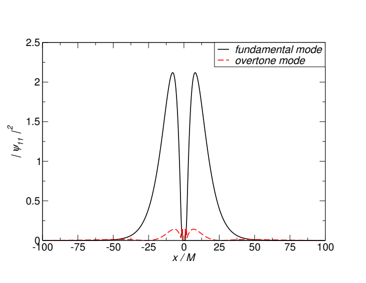
$$\psi \sim A_0 \exp(-\imath\omega_0 t) + A_n \exp(-\imath\omega_n t)$$
$$\sim \exp(-\imath\omega_{R,0} t)\big(B_0 + B_n \cos(\delta t) - \imath B_n \sin(\delta t)\big)$$

Massive scalar field in Kerr - bound states

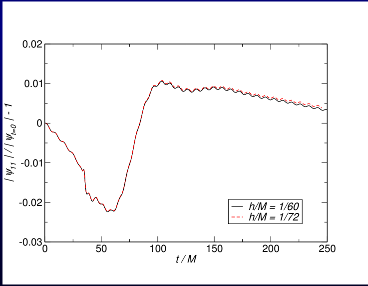
Evolution of bound-state scalar field with $M\mu = 0.42$, $a/M = 0.99$

- localized in the vicinity of the BH
- node of overtone at $x = 26.5\ M$
- by construction $\Psi_{11}\Psi^*_{11} \sim \exp(-\imath\omega t)\exp(\imath\omega t) = const.$

Initial data

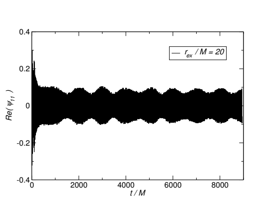


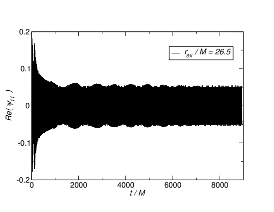
Evolution

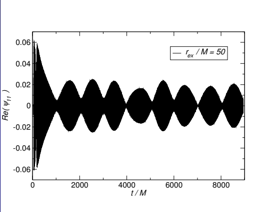


Massive scalar field in Kerr - Gaussian

Evolution of a scalar field with $M\mu = 0.42$ in Kerr $a/M = 0.99$ with Gaussian initial data







- beating of modes if $\omega_{R,n} = \omega_{R,0} + \delta$, $\delta << 1$

$$\psi \sim A_0 \exp(-\imath\omega_0 t) + A_n \exp(-\imath\omega_n t)$$
$$\sim \exp(-\imath\omega_{R,0} t)\big(B_0 + B_n \cos(\delta t) - \imath B_n \sin(\delta t)\big)$$

- amplitude of modes dependent on location of measurement
- possible explanation for result by Strafuss & Khanna '05

Massive vector fields

Massive vector fields

- massive hidden $U(1)$ vector fields from string theory compactification (e.g., Jaeckel & Ringwald '10)
- expected: superradiance effect stronger than in scalar field case
- rich phenomenology

- studied by Galt'sov et al '84, Konoplya '06, Konoplya et al '07, Herdeiro et al '11, *Rosa & Dolan '11*
- vector field eqs. in Kerr non-separable $\Rightarrow$ challenging problem

Massive vector fields in Schwarzschild background

Rosa & Dolan '11:

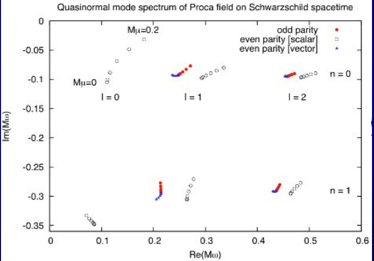
- Proca field equations

$$\nabla_\nu F^{\mu\nu} + \mu_A^2 A^\mu = 0 \quad F_\mu = \nabla_\mu A_\nu - \nabla_\nu A_\mu$$

- Lorenz condition has to be satisfied $\nabla_\mu A^\mu = 0$
  $\Rightarrow$ scalar mode gains physical meaning
- decomposition of A_μ in vector spherical harmonics $Z_\mu^{(i)lm}$
- continued fraction method and forward integration

Massive vector fields in Schwarzschild background

Rosa & Dolan '11: QNM spectrum



- for given l, n:
  2 even parity modes
  (scalar and vector field modes),
  1 odd parity mode (vector field mode)
- in electromagnetic limit ($M\mu_A \to 0$):
  - scalar mode = gauge mode
  - even and odd vector mode degenerate
- field mass - breaking of degeneracy
- distinct frequencies of even parity modes

Massive vector fields in Kerr - Code setup

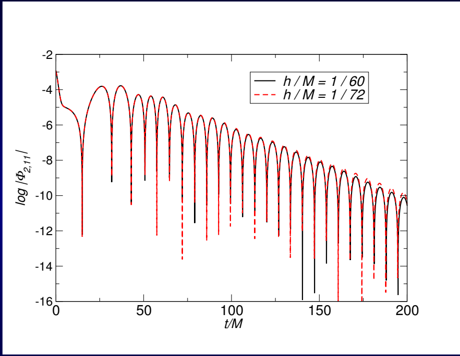
goal: study time evolution of Proca field in Kerr background
(work in progress)

- Kerr background in Kerr-Schild coordinates $\rightarrow$ excision of BH region
- Proca equation $\nabla_\nu F^{\mu\nu} + \mu_A^2 = 0$
  Lorenz condition $\nabla_\mu A^\mu = 0$
- define $A_\mu = \mathcal{A}_\mu + n_\mu \varphi$, $E_\mu = F_{\mu\nu} n^\nu$
  $\Rightarrow$ formulation as $3+1$ time evolution problem
- initial data: gaussian wave packet
- 4^{th} finite differences in space, 4^{th} order Runge-Kutta time-integrator
- extraction of Newman-Penrose scalar Φ_2 at fixed r_{ex}, mode decomposition

$$\Phi_{2,lm}(t) = \int d\Omega \Phi_2(t,\theta,\phi)\, {}_{-1}Y^*_{lm}(\theta,\phi)$$

Massless vector field in Kerr

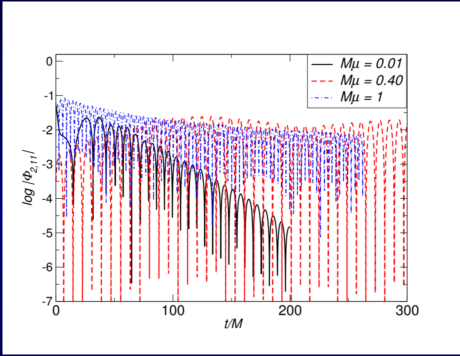
vector field with $\mu_A = 0$ in Kerr with $a/M = 0.99$



- QNM frequencies: $M\omega_{11} = 0.461 - i0.041$ $(0.463 - i0.031)$
- in agreement with theoretical prediction (Berti et al, '09)
- improving with increasing resolution

Massive vector fields in Kerr

- vector field with $\mu_A = 0.01, 0.42, 1$ in Kerr with $a/M = 0.99$



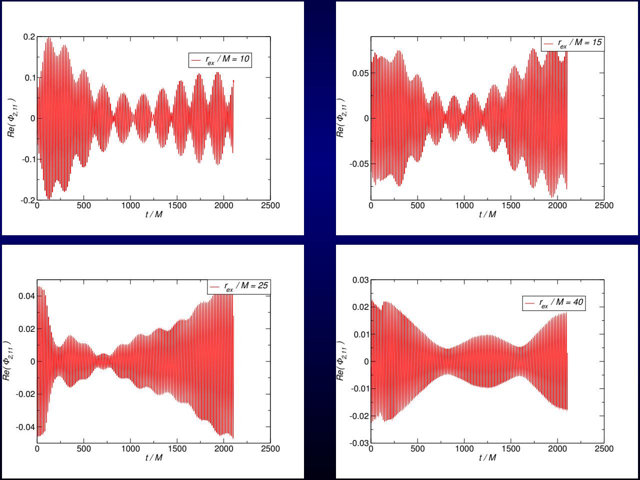
- QNM frequencies

$$
\begin{aligned}
M\mu_A &= 0.01 \quad & \omega_{11} M &= 0.462 - i0.041 \\
M\mu_A &= 0.40 \quad & \omega_{11} M &= 0.415 \\
M\mu_A &= 1.0 \quad & \omega_{11} M &= 0.959 - i0.004
\end{aligned}
$$

Proca field in Kerr

Evolution of a Proca field with $M\mu = 0.40$ in Kerr $a/M = 0.99$

- beating of modes
- growth of the mode - signature of instability?



Conclusions

- massive fields in Kerr spacetimes exhibit extremely rich spectra
- evolution of scalar field wave packets
  - extensive code testing
  - beating of fundamental and overtone modes
- first evolutions of massive vector fields in Kerr background
  - low mass fields $M\mu_A = 0.01$ damped
  - beating effect for $M\mu_A = 0.42$
  - growing of $l = m = 1$ mode $\rightarrow$ possible signature of instability
  - still in its infantry $\Rightarrow$ more results to come soon

Thank you!

http://blackholes.ist.utl.pt