Radiation from a D-dimensional collision of shock waves: numerical methods

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nr/hep^2 - IST

Recap

Yesterday we have seen:

- How to formulate the exact initial value problem for shock wave collision spacetime.
- A perturbative approximation, valid close to the axis at late times, which is a tower of wave equations with sources.
- Reduced the problem to computation of scalar functions

  $$F_m^{(n)}(u,v,\rho) = F_{m,Surf}^{(n)} + F_{m,Vol}^{(n)}$$

  and focused on the surface integrals
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Sensitive features of the integrand $I_m^{D,n}(x_\star)$

Integrand contains several pieces **to be computed with care**:

1. Polynomials $Q_m^{D,n}(x_\star), \ldots$ coefficients defined differentially:
   Example of *task* to be done *once at the beginning.*
   (see Mathematica example 2.1)

2. D odd contains a potentially problematic scaling.
   Example where *asymptotic expansions* can be useful!
   (see Mathematica example 2.2)

3. Singularities @ $x_\star = s = \pm 1$, due to

$$\frac{1}{\sqrt{\pm(1-x_\star^2)}} = \frac{1}{\sqrt{(1-x_\star)(\pm x_\star \pm 1)}}$$

   - Adapted *change of variable* $y = y_s^{\text{root}} - kw^2$ & *split domains*.
   - Writing *expressions* & using *series in problematic regions*.

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Plan - Part II

1. Radiation extraction
   - First order wave signal
   - Bondi formula vs Pseudo-tensor & extraction
2. Higher orders and two dimensional reduction
   - The Volume terms
   - Two dimensional form
   - The 2D Green's functions
3. Summary & Conclusions

Outline

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Visualisation of the radiation signal $D = 4$

Play Video

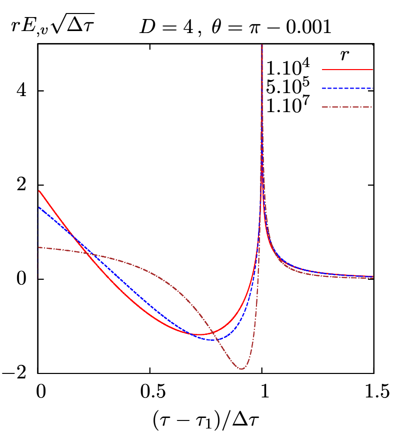
- Signal follows optical rays & geometrical suppression.
- Growth at axis (shorter $\delta\tau$) & far away (pert. breakdown).

Visualisation of the radiation signal $D = 5$

Play Video

Suppressed tail in D-odd after second optical ray.

Reduced Wave forms



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$$C(x) \equiv \frac{2}{E_{CM}} \frac{dE}{dx} = \sum_{n=0}^{+\infty} C_n (x^2 - 1)^n$$

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$$\Rightarrow \text{Inelasticity } \epsilon_{\text{radiated}} = \sum_{n=0}^{+\infty} \frac{C_n (-2)^n n!}{(2n+1)!!}$$

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- **Example:**
  $D = 4$ we know $C_0 \simeq 0.250$ & $C_0 - 2C_1/3 = 0.163$

$$\Rightarrow C_1/C_0 \simeq 0.52$$

Thus: 1^{st} order $\Leftrightarrow$ isotropic approximation, $n = 0$
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Radiation extraction (Landau-Lifshitz pseudo-tensor)

- **Radiative** components $h_{ij} = \delta_{ij} H(u, v, \rho) + \Delta_{ij}(x) E(u, v, \rho)$
- At first order ($i = 1$) only $E(u, v, \rho)$
- In our first paper $\rightarrow$ Landau-Lifshitz pseudo-tensor

  Yoshino and Shibata, arXiv:0907.2760

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E_{\text{radiated}} &= \int dt \int_{S^{D-2}} \frac{dEnergy}{dSdt} \\
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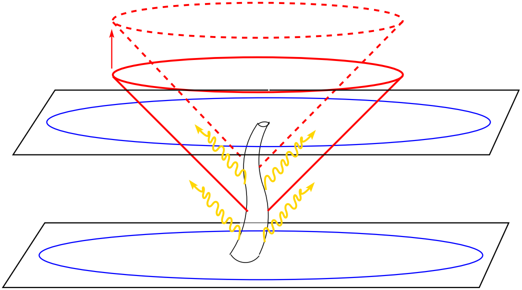
Bondi extraction & Higher orders . . .



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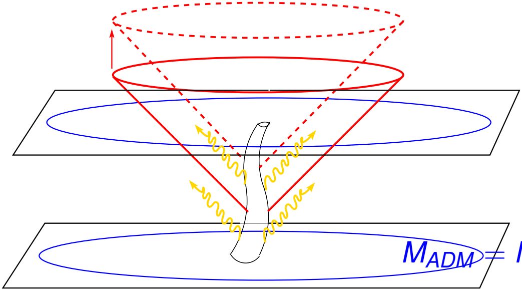

$M_{ADM} = M_{final} + E_{rad}$

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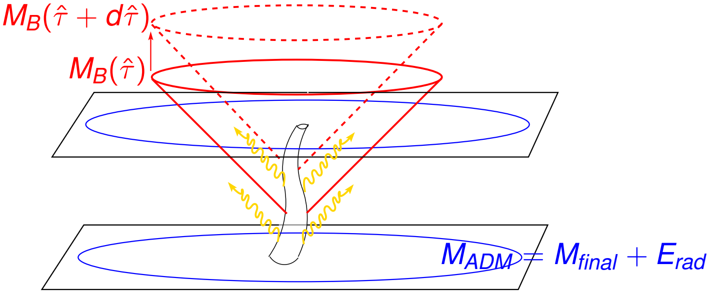

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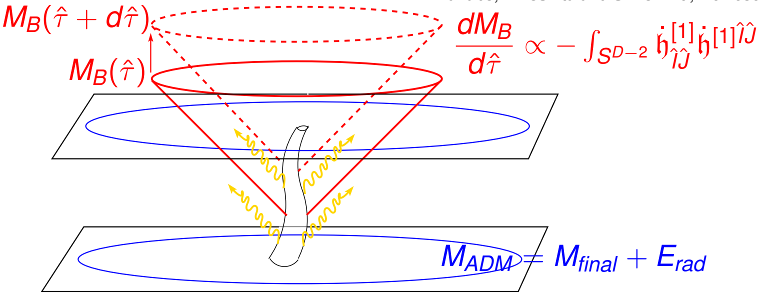
Tanabe, Kinoshita and Shiromizu,1104.0303



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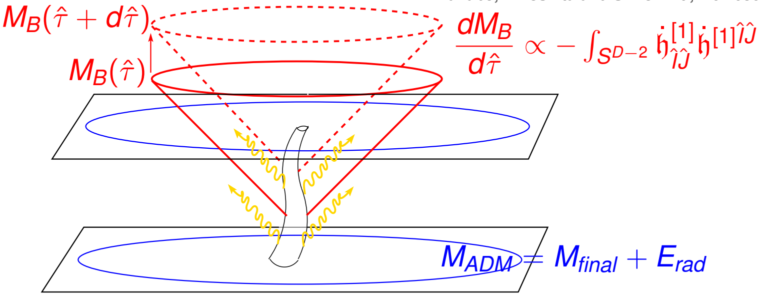

de Donder coordinates $\rightarrow$ **Bondi** coordinates:

$$ds^2 = g_{\hat{\tau}\hat{\tau}} d\hat{\tau}^2 + 2g_{\hat{\tau}\hat{r}} d\hat{\tau} d\hat{r} + 2g_{\hat{I}\hat{\tau}} dx^{\hat{I}} d\hat{\tau} + \hat{r}^2 \left[\omega_{\hat{I}\hat{J}} + \frac{\mathfrak{h}^{[1]}_{\hat{I}\hat{J}}}{\hat{r}^{D/2-1}} + \ldots \right] dx^{\hat{I}} dx^{\hat{J}}$$

- Conditions: $g_{\hat{r}\hat{r}} = g_{\hat{r}\hat{\theta}} = 0$ & $\hat{r}$ areal radius
- **Use** de Donder gauge conditions: $\bar{h}^{\alpha\beta}{}_{,\beta} = 0$

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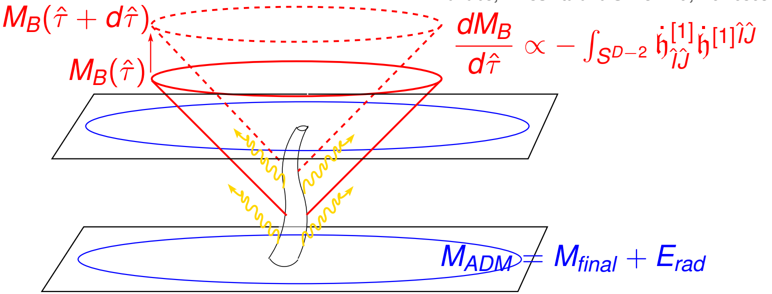

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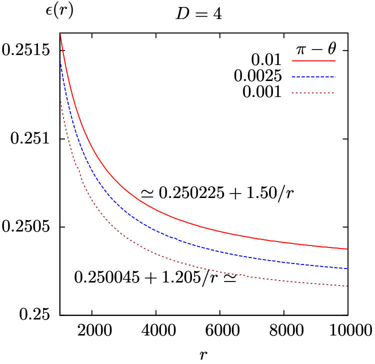
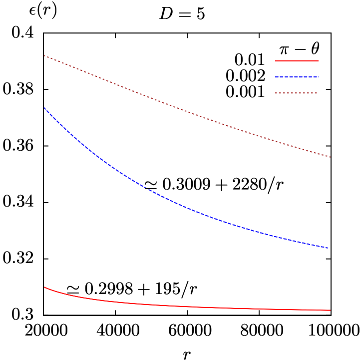
$$\Rightarrow \frac{dM_B}{d\hat\tau\, d\cos\hat\theta} = -\frac{(D-2)(D-3)\Omega_{D-3}}{32\pi G_D}\lim_{\hat r\to+\infty}\left[\hat r\hat\rho^{\frac{D-4}{2}}\left(\dot E + \dot H\right)\right]^2$$

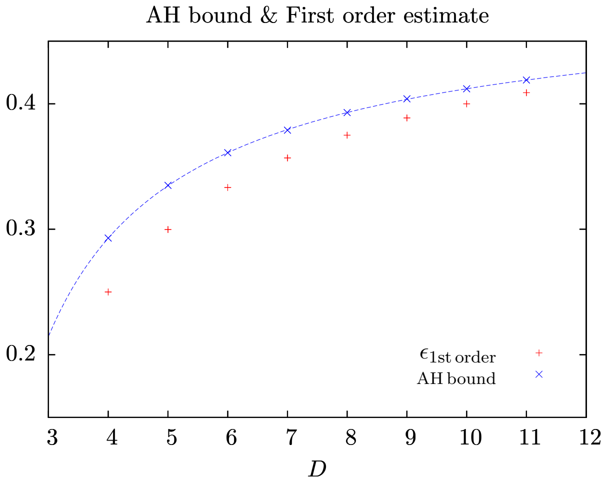
Extracting the inelasticity @ 1st order





D	4	5	6	7	8	9	10
$\epsilon_{\mathrm{1st\,order}}$(%)	25.0	30.0	33.3	35.7	37.5	38.9	40.0
AH bound (%)	29.3	33.5	36.1	37.9	39.3	40.4	41.2

A remarkably simple fit formula



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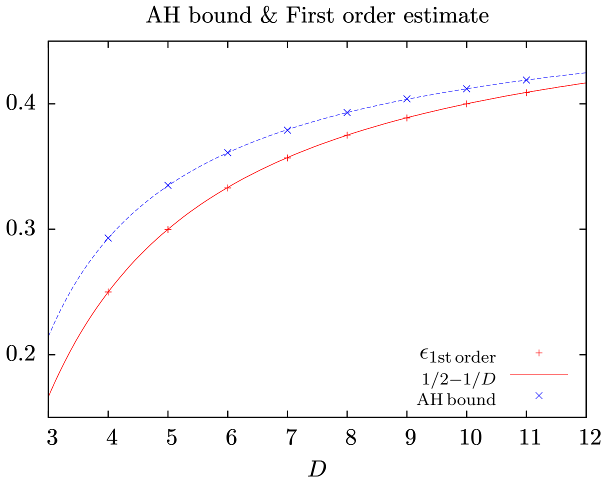

$$\epsilon_{\text{1st order}} = \frac{1}{2} - \frac{1}{D} \quad (!!!)$$

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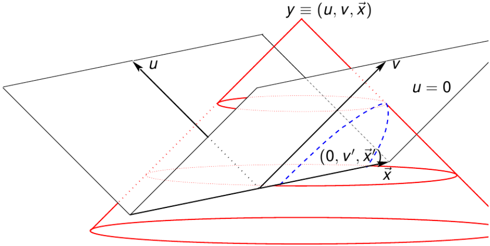
Volume terms & domain conditions

Recall:

$$F_{m,Vol}^{(n)} = \frac{-\Omega_{D-4}}{2(2\pi\rho)^{\frac{D-2}{2}}} \int_0^u du' \iint_{\mathcal{D}_{Vol}} dv' d\rho' \rho'^{\frac{D-4}{2}} S_F^{(n-1)}(u', v', \rho') I_m^{D,0}(x)$$

$$x = \frac{\rho^2+\rho'^2-2(u-u')(v-v')}{2\rho\rho'} \leq 1$$

($I_m^{D,0}$ in handout).



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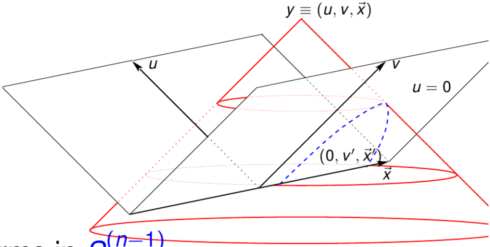

This requires:

- Evaluating $\sim$ 10 surface terms in $S_F^{(n-1)}$
- Triple integration
- Finding a complicated implicit integration domain in 2D
  (Also need to impose above blue surface)

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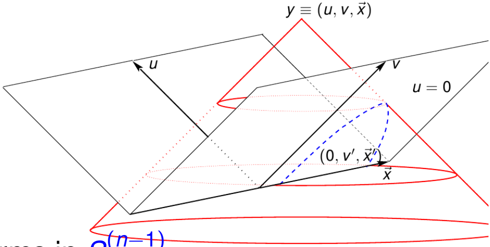

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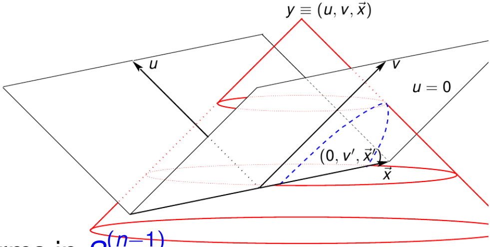

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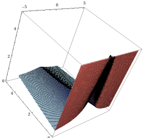

Source functions

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and after a <u>lengthy calculation</u> (see lecture notes)

$$T^{(1)} = -\frac{1}{D-2}(B_{,v})^2 - \frac{2(D-3)}{D-2}\left[\frac{D-3}{\rho}BE_{,v} + B_{,\rho}E_{,v} + BE_{,\rho v}\right] - 2(D-3)E_{,u}E_{,v} +$$

$$+(D-3)\left[\frac{4(D-3)}{\rho^2}E^2 + \frac{D}{2(D-2)}(E_{,\rho})^2 - \frac{D^2-9D+16}{(D-2)\rho}EE_{,\rho} - \frac{D-4}{D-2}EE_{,\rho\rho}\right]$$

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$$T^{(1)}_{ij} = T^{(1)}(u,v,\rho)\delta_{ij} + S^{(1)}(u,v,\rho)\Delta_{ij}$$

and after a lengthy calculation (see lecture notes)

$$T^{(1)} = -\frac{1}{D-2}(B_{,v})^2 - \frac{2(D-3)}{D-2}\left[\frac{D-3}{\rho}BE_{,v} + B_{,\rho}E_{,v} + BE_{,\rho v}\right] - 2(D-3)E_{,u}E_{,v} +$$
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and

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$\Rightarrow$ **Some simplification is in order for feasibility!**

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1 Radiation extraction
- First order wave signal
- Bondi formula vs Pseudo-tensor & extraction

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3 Summary & Conclusions

2D reduction & CL symmetry

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This also **holds** for system of shocks **order by order**.

Thus:

- One finds $D-1$ invariant coordinates $\{p, q, \phi_i\}$
- All ρ dependence scales out!
- $\Rightarrow$ Problem becomes 1+1 in $\{p, q\}$ order by order

(Recall $p \equiv (\sqrt{2}v - \Phi(\rho))\rho^{D-4}$ and $q \equiv \frac{u}{\rho^{D-2}}$)

Integral solutions in 2D form

More specifically, the symmetry implies:

$$h^{(i)}_{\mu\nu}(p,q,\phi_i,\rho) = \rho^{-(D-3)(2i+N_u-N_v)} f^{(i)}_{\mu\nu}(p,q,\phi_i)\ .$$

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And so the previous solutions acquire the form

$$f^{(n)}_{m,Surf}(p,q) = -\frac{n!(-1)^D \Omega_{D-4}}{(2\pi)^{\frac{D-2}{2}}} \left(\frac{\sqrt{2}}{q}\right)^n \int_{\mathcal{D}_{surf}} dy\, f(y)\, y^{\frac{D-4}{2}+n} I^{D,n}_m(x_\star)$$

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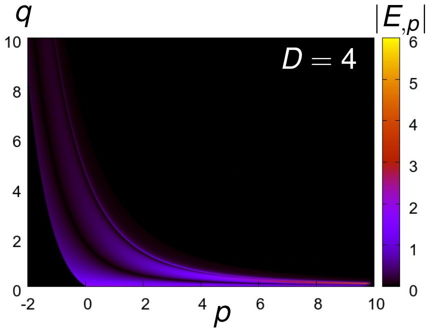
with

$$G^{(n,m)}(p,q,p',q') \equiv \frac{(-1)^{D+1}\Omega_{D-4}}{(2\pi)^{\frac{D-2}{2}}}\int_{\mathcal{D}'_{surf}} dy\, y^{\frac{D-4}{2}-2n(D-3)} I^{D,0}_m(x)$$

$$x = \frac{1+y^2-\sqrt{2}(q-q'y^{D-2})(p-p'y^{-(D-4)}-\psi(y))}{2y} \leq 1$$

$\Rightarrow \hat{C}_-(y) \leq 0$ (Note: When $p' = q' = 0$, $\hat{C}_-(y) \to C_-(y)$)

First order results in 2D form



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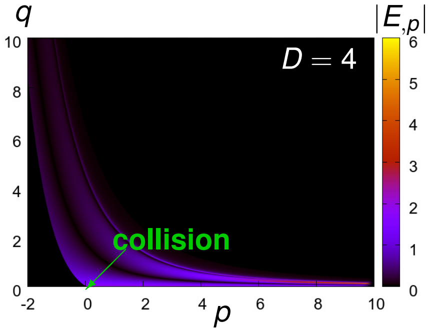

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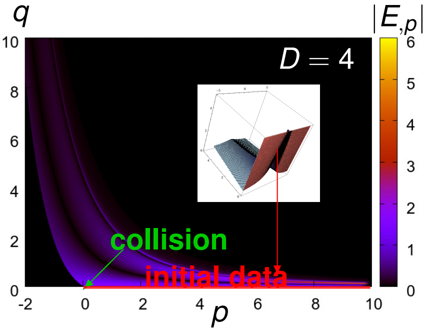

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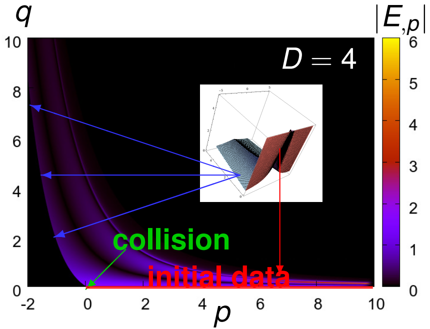

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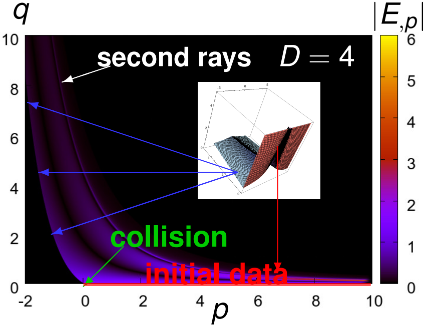

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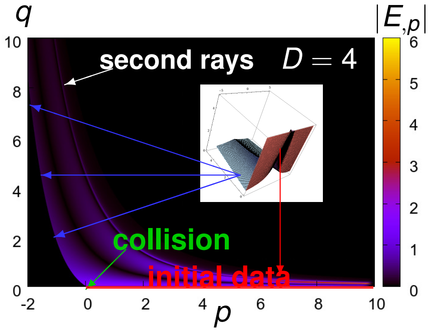

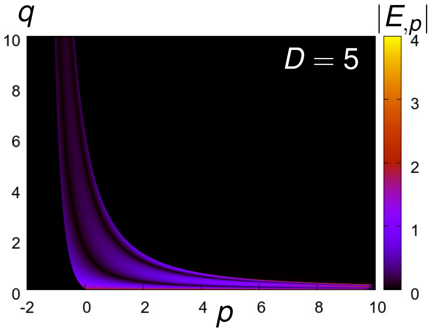

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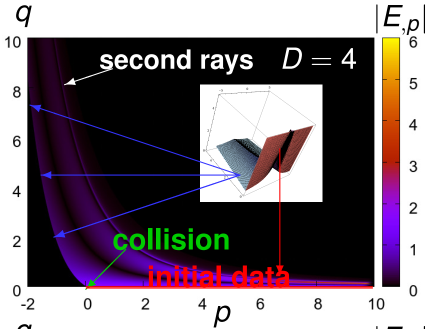

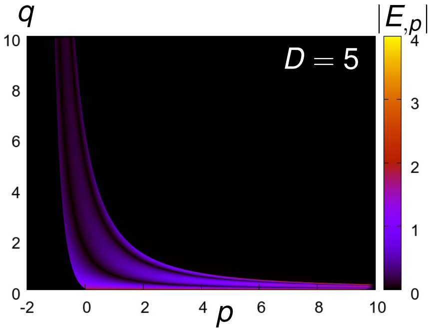

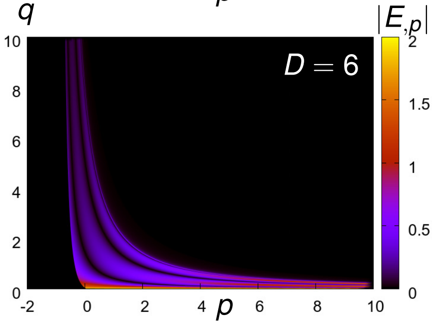

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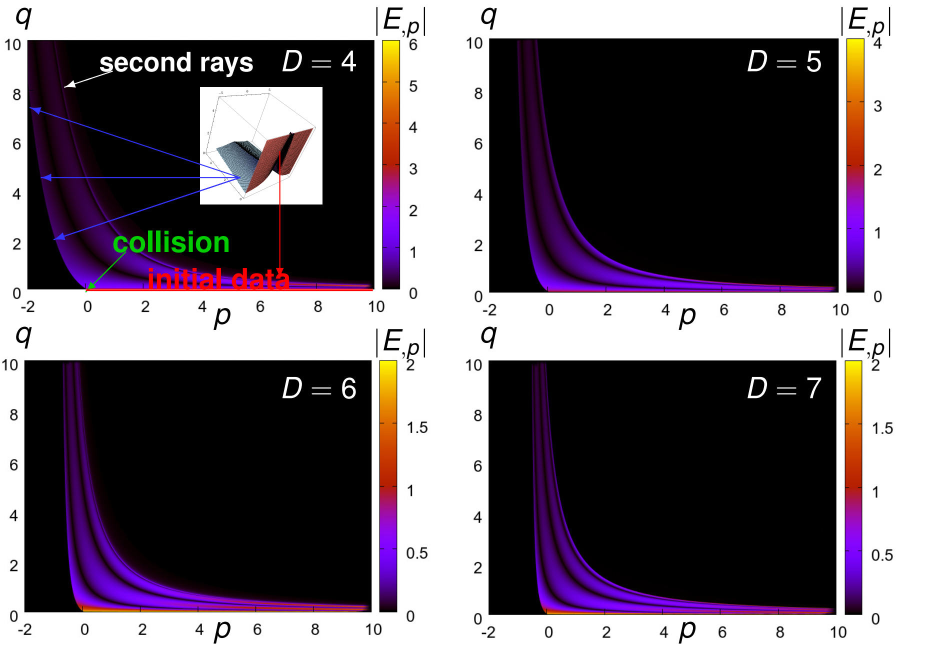

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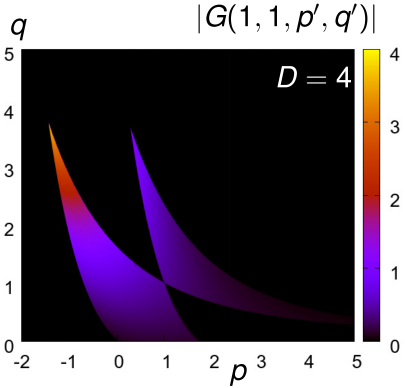
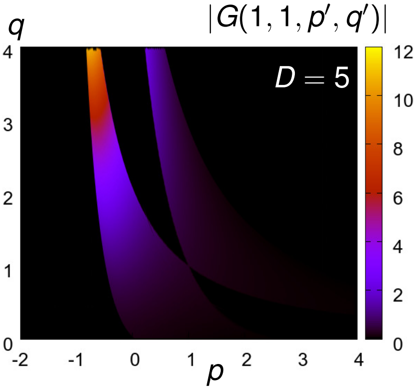
Characteristics & Past light cone visualisation





Domain conditions & generalised domain search

In 2D form the domain condition (defining light-cone) is a more complicated polynomial. **The procedure becomes:**

- Construct a chain of derivatives $C_s^{(k)} \sim \frac{d\hat{C}_s(y)}{dy}, k = 0, \ldots, 3$
- Find root of $C_s^{(3)}$ and use that to find possible root configuration for $C_s^{(2)}$.
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To perform the source (double) integral in the 2D plane, need generalised domain search $\Rightarrow$ 2D bracketing strategy.

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- Inelasticity of BH formation in transplanckian collisions.
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Thanks for your attention!